Examples on Computing Present Value and Yield to Maturity

(Econ 121: Mishkin Chapter 4 Materials)

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A Useful Formula:

$$a + a^2 + a^3 + ... + a^n = \frac{a - a^{n+1}}{1 - a}. \tag{1}$$

Special Case: When $0 < a < 1$, and $n \to \infty$,

$$a + a^2 + a^3 + ... + a^{\infty} = \frac{a}{1 - a}. \tag{2}$$

Example 1 *Calculate the present value for the following payments:*

1. \$500 two years from now when the interest rate is 5%: $\frac{500}{(1+0.05)^2}$.
2. \$100 every three years for 12 years when the interest rate is 10%::

$$\begin{aligned} PV &= \frac{100}{(1+0.1)^3} + \frac{100}{(1+0.1)^6} + \frac{100}{(1+0.1)^9} + \frac{100}{(1+0.1)^{12}} \\ &= 100\left[\frac{1}{1.1^3} + \frac{1}{1.1^6} + ... + \frac{1}{1.1^{12}}\right]. \end{aligned}$$

We can apply the formula in equation (1) by recognizing that $a = \frac{1}{1.1^3}$ *and* $n = 4$ *in this case. Applying the formula, we have*

$$PV = 100 \times \frac{\frac{1}{1.1^3} - \left[\frac{1}{1.1^3}\right]^{4+1}}{1 - \frac{1}{1.1^3}} = 205.85.$$

3. \$100 every three years for 12 years when the interest rate is 10%, plus \$50 bonus at the end of 12 years.

$$\begin{aligned} PV &= \frac{100}{(1+0.1)^3} + \frac{100}{(1+0.1)^6} + \frac{100}{(1+0.1)^9} + \frac{100}{(1+0.1)^{12}} + \frac{50}{(1+0.1)^{12}} \\ &= 221.78 \end{aligned}$$

Example 2 *Suppose you buy a \$1000 face-value coupon bond with a coupon rate of 10%, a maturity of 4 years,*

1. Suppose you purchase the bond at a price of \$1000, what is the yield to maturity?

First write down the formula for yield to maturity:

$$\begin{aligned} 1000 &= \frac{1000 \times 10\%}{(1+i)} + \frac{1000 \times 10\%}{(1+i)^2} + \frac{1000 \times 10\%}{(1+i)^3} + \frac{1000 \times 10\%}{(1+i)^4} + \frac{1000}{(1+i)^4} \\ &\Rightarrow \quad i = 10\% \end{aligned}$$

2. Suppose the purchase price is \$800, what is the yield to maturity?

$$\begin{aligned} 800 &= \frac{1000 \times 10\%}{(1+i)} + \frac{1000 \times 10\%}{(1+i)^2} + \frac{1000 \times 10\%}{(1+i)^3} + \frac{1000 \times 10\%}{(1+i)^4} + \frac{1000}{(1+i)^4} \\ &\Rightarrow \quad i = 17.34\% \end{aligned}$$

3. Suppose the purchase price is \$1200, what is the yield to maturity?

$$\begin{aligned} 1200 &= \frac{1000 \times 10\%}{(1+i)} + \frac{1000 \times 10\%}{(1+i)^2} + \frac{1000 \times 10\%}{(1+i)^3} + \frac{1000 \times 10\%}{(1+i)^4} + \frac{1000}{(1+i)^4} \\ &\Rightarrow \quad i = 4.43\% \end{aligned}$$